Fractional integral equations with weighted Takagi-Landsberg functions

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In the talk, we find fractional Riemann-Liouville derivatives for the Takagi-Landsberg functions. Moreover, we introduce their generalizations called weighted Takagi-Landsberg functions. Namely, for constants $c_{m,k} \in [-L, L]$, $k, m \in \mathbb{N}_0$, we define a weighted Takagi-Landsberg function as $y_{c,H} : [0, 1] \to \mathbb{R}$ via

$$
y_{c,H}(t) = \sum_{m=0}^{\infty} 2^m \left( \frac{1}{2} - H \right) 2^{m-1} \sum_{k=0}^{2^m-1} c_{m,k} e_{m,k}(t), \quad t \in [0, 1],$$

where $H > 0$, $\{e_{m,k}, m \in \mathbb{N}_0, k = 0, \ldots, 2^m - 1\}$ are the Faber-Schauder functions on $[0,1]$. The class of the weighted Takagi-Landsberg functions of order $H > 0$ on $[0,1]$ coincides with the $H$-Hölder continuous functions on $[0,1]$. Based on computed fractional integrals and derivatives of the Haar and Schauder functions, we get a new series representation of the fractional derivatives of a Hölder continuous function. This result allows to get the new formula of a Riemann-Stieltjes integral. The application of such series representation is the new method of numerical solution of the Volterra and linear integral equations driven by a Hölder continuous function.