Fractional integral equations with weighted Takagi-Landsberg functions

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In the talk, we find fractional Riemann-Liouville derivatives for the Takagi-Landsberg functions. Moreover, we introduce their generalizations called weighted Takagi-Landsberg functions. Namely, for constants $c_{m,k} \in [-L, L], k, m \in \mathbb{N}_0$, we define a *weighted Takagi-Landsberg function* as $y_{c,H}: [0,1] \to \mathbb{R}$ via

$$y_{c,H}(t) = \sum_{m=0}^{\infty} 2^{m\left(\frac{1}{2}-H\right)} \sum_{k=0}^{2^m-1} c_{m,k} e_{m,k}(t), t \in [0,1],$$

where H > 0, $\{e_{m,k}, m \in \mathbb{N}_0, k = 0, \ldots, 2^m - 1\}$ are the Faber-Schauder functions on [0,1]. The class of the weighted Takagi-Landsberg functions of order H > 0 on [0,1] coincides with the *H*-Hölder continuous functions on [0,1]. Based on computed fractional integrals and derivatives of the Haar and Schauder functions, we get a new series representation of the fractional derivatives of a Hölder continuous function. This result allows to get the new formula of a Riemann-Stieltjes integral. The application of such series representation is the new method of numerical solution of the Volterra and linear integral equations driven by a Hölder continuous function.