PROPERTIES OF UTILITY MAXIMIZATION FUNCTIONALS FOR NON-CONCAVE UTILITY FUNCTION IN COMPLETE MARKET MODEL

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This work is devoted to the study of the utility maximization problem. There are a lot of different aspects which can be considered while solving the optimization problem, such as completeness of the market, properties of the utility function, model settings, modeling of the payoff, and so on. We consider the complete market model, non-decreasing upper-semicontinuous non-concave utility function satisfying mild growth condition, and study the standard and constrained optimization problems while considering both the standard and robust utility maximization problems.

We proved the existence and uniqueness of the optimal solution to the standard non-concave utility maximization problem and constructed its explicit form under the assumption of standard budget constraints. It was shown that this solution is also a unique optimal solution for the maximization problem of the concavified utility function.

In the case of implementing an additional upper bound given by some random variable, we proved a similar theorem if the given random variable is discrete. Moreover, we presented examples that show that previous conclusions may fail in the case of a continuous random variable that represents an upper bound.

Subsequently, in the case of a robust utility maximization problem deriving the optimal solution is based on the study of the minimax identity for the initial non-concave utility function. We obtained equalities and inequalities to relate the robust utility functional of the initial utility function and its concavification and derived the assumptions under which minimax identity holds for the initial utility function. Besides, similar results were obtained in the case with an additional upper bound on the budget, represented, as before, by some random variable. The crucial step for obtaining the mentioned results with implementing an additional upper bound is the use of the regular conditional distribution which sheds new light on the possible approaches for solving the optimization problem.

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